

# **A Methodological Approach for the Valuation of Callable Bonds in Emerging Markets: The TGI example<sup>1</sup>**

***Edgardo Cayón Fallon<sup>2</sup> Y Julio Sarmiento Saboga<sup>3</sup>***

***CESA (Colegio de Estudios Superiores de Administración) and Pontificia Universidad Javeriana)***

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<sup>2</sup> MBA McGill University, Montreal, Canadá, 2001; BS Economics and Finance, Syracuse University, 1995. Profesor Asociado en Finanzas. Colegio de Estudios Superiores de Administración (CESA), Correo electrónico: [ecayon@cesa.edu.co](mailto:ecayon@cesa.edu.co)

<sup>3</sup> Especialista en Gerencia Financiera de la Pontificia Universidad Javeriana, 2001; administrador de empresas de la Pontificia Universidad Javeriana, 1998. Profesor, Departamento de Administración, Facultad de Ciencias Económicas y Administrativa, Pontificia Universidad Javeriana. Coordinador académico, especialización en Gerencia Financiera, FCEA Pontificia Universidad Javeriana. Correo electrónico: [sarmien@javeriana.edu.co](mailto:sarmien@javeriana.edu.co)

## **ABSTRACT**

The purpose of this paper is to clarify some of the difficulties that a practitioner may find in implementing the binomial model for valuing a corporate bond with multiple embedded options in emerging markets. Especially, when faced with the dilemma of determining which should be the proxy variables for the risk-free rate, sovereign risk and country specific risk. In order to clarify some of the challenges that the practitioners face, the paper will present the reader a practical example that can serve as a guide through the required steps needed to value a callable bond in an emerging market. The callable bond used in this example is issued by the Transportadora de Gas del Interior International Ltd., which is a company located in Colombia and its economic activity is the transportation of natural gas and has four embedded call options by the issuer until its final maturity in October 3, 2017. Our conclusion is that by using the binomial model to find the option adjusted spread of the bond is also possible to find a more reliable measure of specific or unique risk attributable to the company economic activities.

## **INRODUCTION**

As opposed to equities, and setting the issue credit quality aside, the pricing of bonds depends solely on the future behavior of interest rates and their effect in discounting future expected cash flows. In bonds, with embedded calls from the issuer, this represents a distinct challenge because the issuer can alter the nature of the cash flows that the investor will perceive depending on the future behavior of interest rates. Therefore, the investor faces the risk of prepayment from the part of the issuer at a lower rate than the one that the investor is currently receiving for holding the bond until maturity (Rubio 2005). Since the investor faces the risk of an uncertain stream of cash flows, the common market practice is to demand a higher yield in a callable bond than in a not callable bond in order to compensate the higher risk caused by the embedded call options in a specific issue. In common practice, the credit and liquidity risk of any common not callable bond is determined by the additional yield spread paid by that bond when compared to the yield of a risk-free bond with a similar maturity date (i.e. Corporate Issues vs. U.S. Treasuries). In the case of callable bonds the additional spread demanded by the investor above the credit and liquidity risk premium is known as the Option Adjusted Spread (OAS). In order to calculate the OAS, assumptions have to be made about the behavior of the volatility of the bonds and its effect on future yields, and therefore model risk is a factor that has to be taken into account when valuing callable bonds (Henderson 2003). In the US numerous studies have been conducted regarding the behavior of the OAS of callable vs. non callable bonds. For example, Longstaff (1992) found that the implicit call values in callable US treasuries are sometimes overpriced in comparison to their theoretical value due to negative option values. This claim was later contested by Edleson et al. (1993) which demonstrated that the apparent mispricing was not caused by negative option values, but other factors

attributable to other risk factors. Dolly (2002) found that in average the call value of US corporate callable bonds during the period comprehended between 1973 and 1994 was 2,25% of par, and that the price patterns are consistent with the ones that one should expect from commonly used option pricing models. In the specific case of TGI, there is an additional risk factor that has to be taken into account which is country risk. The problems that an investor faces with sovereign risk are not easy to handle because there are a series of factors than can affect the spread attributable to this specific kind of risk. For example Eichengreen and Moody (1999) found that market sentiment was instrumental in determining emerging market spreads in 1996-1994. Also, according to Erb et al. (1999), one the greatest challenges in emerging markets bond valuation is the nature of the term structure of interest rates. Given the fact that in times of crisis, their returns are highly correlated with those of emerging market equities, this generates tracking errors that alter the nature of the term structure of interest rates in those markets over certain periods of time. This means that when dealing with emerging market issues, such as the one used as an example in this paper, one has to be careful in using models that really capture the short and long term volatility that affect interest rates relevant to the debt issue under scrutiny. Finally, our specific objective is to demonstrate via a practical example how the binomial pricing model can be used to determine the OAS and the specific risk of a callable bond of issued by a company that is located in an emerging market by using a market based approach in incorporating the company's country risk spread.

### **THE BINOMIAL PRICING MODEL: A SIMPLE APPROACH FOR VALUING EMBEDDED OPTIONS IN CALLABLE BONDS**

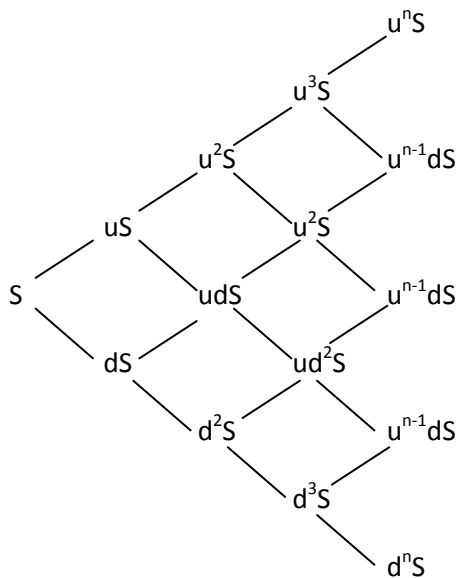
According to Rubio (2005) when valuing callable bonds it is preferable to use the binomial pricing model rather than the Black-Scholes model. This is because Black and Scholes incorporate the following assumptions in the model that most of the time does not apply to bonds and the term structure of interest rates in general:

1. Black and Scholes assume that interest rates are constant trough the life of the bond, this assumption is not realistic since all bonds have reinvestment risk, except in the case of zero-coupon bonds.
2. Black and Scholes assume a infinite lognormal price distribution which is the case for stocks, but not for bonds since the later have a known time to expiration.
3. Constant volatility trough the period of valuation, which in the specific case of bonds in not just a function of price, but is a function of variability in interest rates that tend to change over time as the bond reaches expiration.

The binomial model as proposed by Cox-Ross-Rubinstein (1979) offers the following advantages over Black-Scholes when valuing callable bonds. The reason being, that even though closed-form option pricing models (i.e. Black and Scholes) are easier to handle, those models do not capture many of the features required to value a callable bond. Specially, the Black-Scholes model is extremely inaccurate in capturing the

variations of interest rates throughout the life of the option as well as the embedded value of multiple call options after the first settlement date. Although in practice, when a Binomial Model is taken to the “limit” its results tend to converge with those obtained by Black and Scholes, this occurs because the Binomial Model is simply a discrete approximation of the underlying stochastic differential equation used in Black and Scholes. Given that the Binomial Model distinctive feature is the use of discrete periods, this feature is what gives the Binomial Model a certain advantage over Black and Scholes in the specific case of valuing multiple embedded options in callable bonds. This is because the model assumes (in the specific case of bonds) that the yield of the security evolves on step to step basis as times goes on (Wong 1993). The Binomial Pricing Model assumes that the underlying asset price or yield evolves in a multiplicative binomial pattern in the following manner:

Any node for the price (P) in the lattice tree should go up by an upward factor (u) with a probability (P) or by a downward factor (d) with a probability (1-P) for multiple periods in the following manner:



*Adapted from Lamothe and Perez (2003) p. 88*

In a similar manner we value the intrinsic price of the call option at each node of the lattice using the following formula<sup>4</sup>:

$$C_{t-1} = \frac{1}{(1 + r_f)} \times (p \times C_{tu} + (1 - p) \times C_{td})$$

In which  $C_{t-1}$ =Call value for the preceding period

<sup>4</sup> For a complete development of the algebraic process necessary for finding risk-neutral probabilities and the theoretical background of the principles behind the replicating portfolio inherent in the binomial option pricing formula, we recommend the book “*Opciones Financieras y Productos Estructurados*” by Prosper Lamothe Fernández and Miguel Pérez Somalo, pg. 79-90 (McGraw Hill, 2003

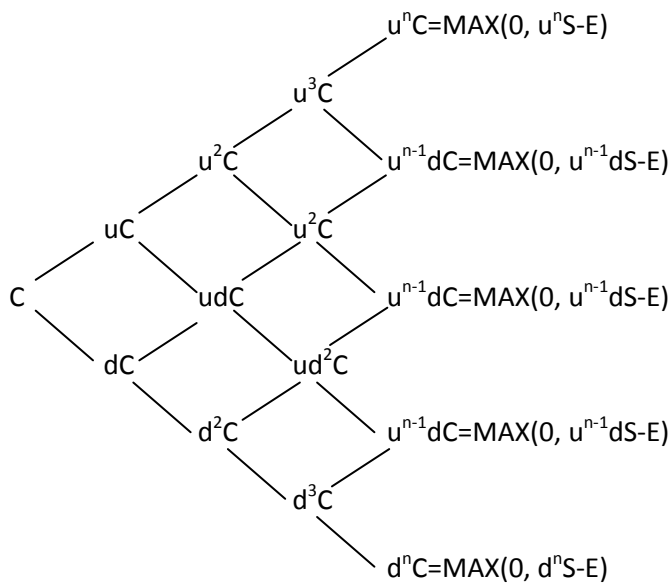
$R_f$ =The proxy variable for the theoretical risk-free interest rate for a given period

$C_{u}$ =The call value for the immediately posterior upward node

$C_{d}$ =The call value for the immediately posterior downward node

$P = \frac{(1+r_f) - d}{u - d}$  or the risk-neutral probability of an upward movement of a replicating portfolio (short or long in a call option, or long or short in risk free bond) where (u) is an upward factor and (d) is a downward factor.

In order to find the option value at each node, the formula is applied a backward way in each node of the following lattice (based on the nominal value obtained for the option of each node at its time of expiration):



Where E is the strike price of the option being valued at a specific point in time, if any value of S is greater than E at the time of expiration the option will be exercised otherwise its value will be zero (0).

*Adapted from Lamothe and Perez (2003) p. 90*

Therefore this approach can be used in valuing multiple embedded options, because by using a lattice we can determine the probability that a call option will be exercised at its expiration date. If indeed, the option is not exercised at a specific node, this means that there is a probability that those cash flows will remain until the next option in the theoretical call schedule expires. By doing this in a repetitive manner, all the calls scheduled in the callable bond will be incorporated into the valuation model. In this way is possible to determine the value of each call embedded on the bond, and how the values of these calls affect the price of the bond and its required yield at a specific point in time.

## A SIMPLE METHODOLOGICAL APPROACH FOR IMPLEMENTING THE BINOMIAL OPTION PRICING MODEL FOR VALUING CALLABLE BONDS: THE TGI EXAMPLE

The main problem faced in option valuation is the one of finding the adequate proxy variables to be used as inputs of the model. Therefore, the main objective of this paper is to use a practical example on the steps required to value a callable bond using the binomial pricing model. In order to develop a meaningful example of how to develop the binomial pricing model, the example will be focused on the valuation of a recent issue by TGI International Ltd. which is a subsidiary of a Colombian company called Transportadora de Gas del Interior, a local monopoly whose business is the transportation and wholesale distribution of Natural Gas. The issue has the following characteristics (Note: For the purpose of this example the valuation date is March 31<sup>st</sup>, 2008):

Issuer:	TGI INTERNATIONAL LTD
Country:	Colombia
Maturity:	October 3, 2017
Coupon:	Fixed 9,5% Semi Annual
Day Count:	30/360
Fitch Rating:	BB
Yield (3/31/2008):	8,872%

Source: Bloomberg

The issue has four embedded call options from the issuer and its call schedule is as follow (it is important to recall that in any coupon payment date the clean price is equal to the dirty price):

Date (mm/dd/yyyy)	Exercise Price
10/03/2012	104.75
10/03/2013	103.167
10/03/2014	101.583
10/03/2015	100

Source: Bloomberg

Some of the problems on how to obtain meaningful proxy variables in order to value this specific issue are the following:

1. Finding a proxy for the risk-free rate given the fact that even though the issue is dollar denominated, the company in question is not US based.
2. Finding a proxy for the volatility of the yield of the proxy used as a risk-free rate that incorporates the additional spread required for country risk.
3. Finding a proxy for a not callable bond issue with the same coupon and maturity date comparable to the issue that is being valued.
4. Finding the spread attributable to specific industry risk.

Therefore, in order to provide a meaningful insight on how to address these issues, a detailed step by step methodological approach is described in the process required to value TGI callable bond issue throughout this paper.

**Step 1-Sovereign Colombian Bonds yield as a proxy variable that incorporates the additional spread required by country risk.**

Before implementing the lattice approach for predicting the behavior of future yields for the specific case of TGI, it was necessary to find a proxy for a not callable bond with the same coupon and maturity dates of TGI. Since, TGI is located in an emerging market there are no comparable issues from a not callable bond in order to determine the OAS of TGI. Therefore, in order to have a meaningful proxy for a not callable bond a synthetic theoretical not callable bond series was created in order to find a meaningful yield that incorporated both the risk-free rate as well as spread attributable to country risk<sup>5</sup>. This theoretical yield was found via linear interpolation using two Colombian Sovereign issues with a maturity date before and after TGI maturity date. The issues have the following characteristics:

Issuer:	REPUBLIC OF COLOMBIA
Country:	Colombia
Maturity:	January 27, 2017
Coupon:	Fixed 7,375% Semi Annual
Day Count:	30/360
Fitch Rating:	BB+
Yield (3/31/2008):	5,803%

Issuer:	REPUBLIC OF COLOMBIA
Country:	Colombia
Maturity:	February 25, 2020
Coupon:	Fixed 11,75% Semi Annual
Day Count:	30/360
Fitch Rating:	BB+
Yield (3/31/2008):	6,091%

Source: Bloomberg

Therefore, the time left to maturity for the Sovereign Bonds expressed in years<sup>6</sup> are 8,82222 and 11,90 respectively, also the time left to maturity for expressed in years for TGI is 9,50555. Since we know the yield to maturity and the time left to maturity of both bonds, we can use a simple interpolation formula to find the theoretical yield of a

<sup>5</sup> In other words, a yield that incorporates the required country risk spread over a US treasury with similar maturity.

<sup>6</sup> To obtain the exact time from the 31<sup>st</sup> of March 2008 until the date of maturity, we first calculate the time left in a semiannual basis (S/A basis), this is done in order to take into account all the coupons left as well as the principal. Then we express the time in an annual basis, because the yields are expressed by the market in an annual basis. Also the fraction is to denote the time left from the current date until the next coupon payment. In the specific case of TGI, in a semiannual basis, this fraction is expressed as 0,0166667. That gives us in total 19,0166667 semiannual periods that divided by two gives us 9,508333 years.

Sovereign Colombian bond that pays a 9,5% fixed semiannual coupon and matures on October 3, 2017 in the following way:

$$5,867\% = 5,803\% + ((9,508333 - 8,82222) \times (6,091\% - 5,803\%) / (11,90 - 8,2222))$$

In this way, we find that the theoretical yield for a Sovereign Colombian Bond with the same maturity date as TGI would be approximately 5,867%.

**Step 2-Theoretical Sovereign Colombian Bonds yield as a proxy variable for volatility estimates.**

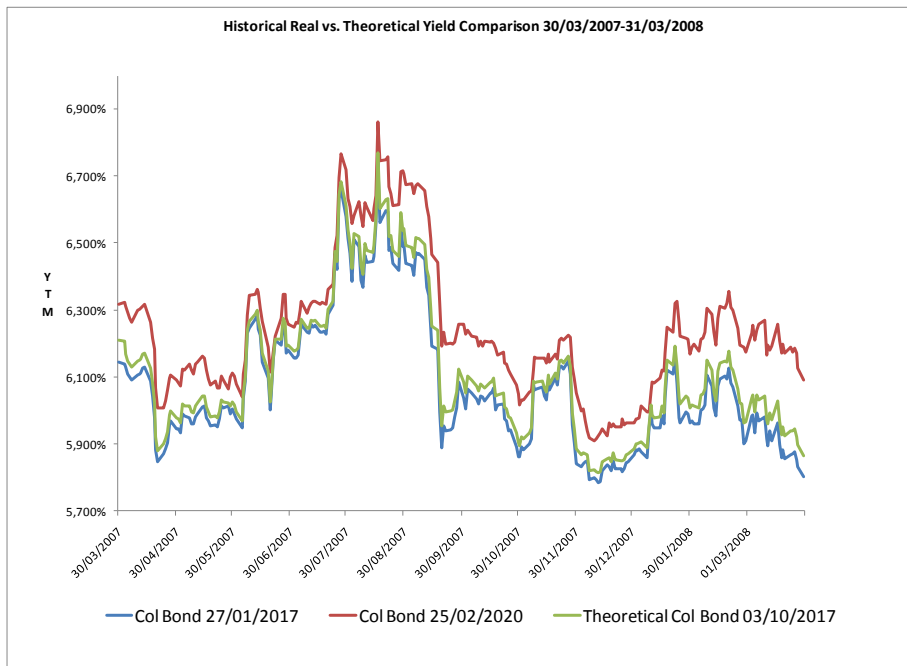
Once we found the approximate theoretical yield of a not callable Sovereign Colombian Bond, we can use the same process for creating a synthetic historical series in order to measure the behavior of the volatility of that theoretical bond in the past. The dataset<sup>7</sup> for obtaining the theoretical yields was conformed of the historical closing prices and yield observations of the 2,017 and 2,020 Colombian Sovereign issues from March 30, 2007 until March 30, 2008. Using simple linear interpolation a theoretical yield was found for each observation that comprised the dataset. Once the yield was obtained, we found the clean price of the theoretical bond for each date. The summary of the historical price and yield behavior for the two sovereign bonds as well as the theoretical bond are compared in exhibit 1 and 2:

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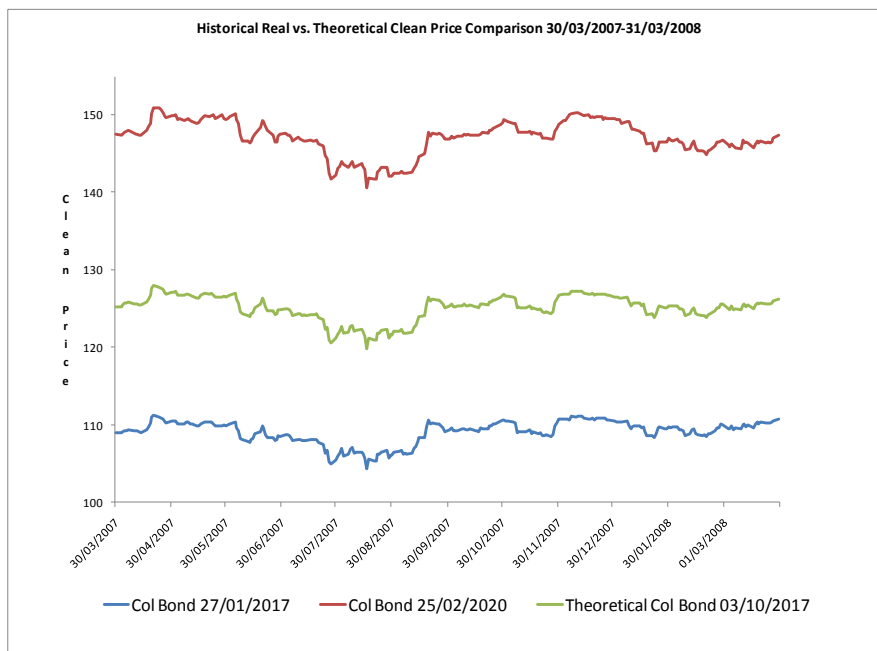
<sup>7</sup> Each dataset was comprised of 262 observations. *Source: Bloomberg.*



### Exhibit 1



### Exhibit 2



Since the yield is the determinant of price in a bond, we proceed to calculate the volatility of the yield of the theoretical bond in the following way, assuming that the yields are continuously compounded:

Daily yield variation is found using the following formula:

$$Y\% = \ln\left(\frac{Y_t}{Y_{t-1}}\right)$$

Once we found the daily yield variations, we can calculate the daily volatility measured by standard deviation using the following formula:

$$\sigma = \sqrt{\frac{1}{n} \sum_{n=1}^n (Y\% - \bar{Y}\%)^2}$$

Where n is the number of observations in the dataset and  $\bar{Y}\%$  is the average daily volatility.

For our specific example our daily volatility is equal to 0,773867%, since the effective trading days for the bonds were 262 and assuming constant volatility we can transform our daily volatility to annual volatility in the following way:

$$\sigma_{year} = \sigma_{daily} * \sqrt{262}$$

Therefore our annual standard deviation is 12,52613% in the way we can obtain the semiannual volatility in the following way:

$$\sigma_{semiannual} = \sigma_{year} * \sqrt{1/2}$$

The semiannual volatility for our theoretical sovereign bond would be 8,85731%, also because we know that there are 3 days for the next semiannual coupon in the TGI case using the same formula we find that the expected volatility for the next three days is equal to 1,34038%.

**Step 3-Constructing a lattice using the Theoretical Sovereign Colombian Bonds yield data and observed volatility.**

If for purposes of simplicity we assume that the yields follow a log-normal distribution (because as prices, the yields can never be below zero), then the upward factor required to construct the lattice would be the geometric standard deviation of the synthetic series or ( $\sigma^e$ ), likewise the downward factor will be the inverse mean or ( $1/\sigma^e$ ), of course this approach for determining the factors assumes that there is not significant variation on the median yield over the life of the option (an assumption that is often violated in practice). Also, a more practical approach would be to use a subjective upward and downward factor based on our feelings toward the behavior of the market for the period under scrutiny (Wong 1993).

Therefore, by applying the formula for the geometric standard deviation in our previous results, we can find the expected semiannual and three days volatility for theoretical issue, the results are:

Yield Volatility Theoretical Bond (Fractional):	1,34038%.
Upward factor:	1,01349379
Downward factor:	0,98668587

Yield Volatility Theoretical Bond (Semi Annual):	8,85731%
Upward factor:	1,09261262
Downward factor:	0,91523746

Using the upward and downward factors we can construct the lattice using as a starting point our semiannual theoretical yield of  $(5,867\%/2)=2,934\%$ . Since the date of the valuation is March 31<sup>st</sup>, 2008 and the next coupon date is April 3<sup>rd</sup>, 2008 the upward and downward expected yields for that specific date in the lattice would be  $2,934\% \times 1,01349379=2,973\%$  and  $2,934\% \times 0,98668587=2,93364^8\%$  respectively. For the dates of October 3<sup>rd</sup>, 2008 onwards we use the semiannual factors using our previous yields in the lattice. Therefore for that specific date the yields are  $2,973\% \times 1,09261262=3,2486\%$  and  $2,973\% \times 0,91523746=2,7212\%$  for the upward branches, for the downward branches the results are  $2,93364\% \times 1,09261262=2,7212\%$  and  $2,93364\% \times 0,91523746=2,6850\%$ . The summary of the results are shown in Table 1:

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<sup>8</sup> The results in the lattice are rounded up to three decimal places, so 2,93364% would be presented as 2,934% in the lattice.

**Table 1-Theoretical Yield Lattice**

Fractionate Semiannual Periods

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

Semiannual Rates

2,934%	2,973%	3,2486%	3,5494%	3,8782%	4,2373%	4,6298%	5,0585%	5,5270%	6,0389%	6,5982%	7,2092%	7,8769%	8,6064%	9,4035%	10,2743%	11,2259%	12,2655%	13,4015%	14,6426%
	2,934%	2,7212%	2,9732%	3,2486%	3,5494%	3,8782%	4,2373%	4,6298%	5,0585%	5,5270%	6,0389%	6,5982%	7,2092%	7,8769%	8,6064%	9,4035%	10,2743%	11,2259%	12,2655%
		2,6850%	2,4905%	2,7212%	2,9732%	3,2486%	3,5494%	3,8782%	4,2373%	4,6298%	5,0585%	5,5270%	6,0389%	6,5982%	7,2092%	7,8769%	8,6064%	9,4035%	10,2743%
			2,4574%	2,2794%	2,4905%	2,7212%	2,9732%	3,2486%	3,5494%	3,8782%	4,2373%	4,6298%	5,0585%	5,5270%	6,0389%	6,5982%	7,2092%	7,8769%	8,6064%
				2,2491%	2,0862%	2,2794%	2,4905%	2,7212%	2,9732%	3,2486%	3,5494%	3,8782%	4,2373%	4,6298%	5,0585%	5,5270%	6,0389%	6,5982%	7,2092%
					2,0585%	1,9094%	2,0862%	2,2794%	2,4905%	2,7212%	2,9732%	3,2486%	3,5494%	3,8782%	4,2373%	4,6298%	5,0585%	5,5270%	6,0389%
						1,8840%	1,7476%	1,9094%	2,0862%	2,2794%	2,4905%	2,7212%	2,9732%	3,2486%	3,5494%	3,8782%	4,2373%	4,6298%	5,0585%
							1,7243%	1,5994%	1,7476%	1,9094%	2,0862%	2,2794%	2,4905%	2,7212%	2,9732%	3,2486%	3,5494%	3,8782%	4,2373%
								1,5781%	1,4639%	1,5994%	1,7476%	1,9094%	2,0862%	2,2794%	2,4905%	2,7212%	2,9732%	3,2486%	3,5494%
									1,4444%	1,3398%	1,4639%	1,5994%	1,7476%	1,9094%	2,0862%	2,2794%	2,4905%	2,7212%	2,9732%
										1,3219%	1,2262%	1,3398%	1,4639%	1,5994%	1,7476%	1,9094%	2,0862%	2,2794%	2,4905%
											1,2099%	1,1223%	1,2262%	1,3398%	1,4639%	1,5994%	1,7476%	1,9094%	2,0862%
												1,1073%	1,0271%	1,1223%	1,2262%	1,3398%	1,4639%	1,5994%	1,7476%
													1,0135%	0,9401%	1,0271%	1,1223%	1,2262%	1,3398%	1,4639%
														0,9276%	0,8604%	0,9401%	1,0271%	1,1223%	1,2262%
															0,8489%	0,7875%	0,8604%	0,9401%	1,0271%
																0,7770%	0,7207%	0,7875%	0,8604%
																	0,7111%	0,6596%	0,7207%
																		0,6508%	0,6037%
																			0,5957%

#### **Step 4-Finding a theoretical discounted not callable sovereign bond price lattice using the future expected yield behavior lattice.**

The first step for finding the discounted not callable sovereign bond price is to calculate the risk-neutral probabilities for a replicating portfolio at each node. The upward and downward risk neutral probabilities are found using the semiannual and three days observed theoretical rate of 2,934% and 0,049%=(2,934% x 3/180) as follows:

$$\text{Upward risk-neutral semiannual probability} = (1 + 2,934\% - 0,91523746) / (1,09261262 - 0,91523746) = 64,326\%$$

$$\text{Downward risk-neutral semiannual probability} = 1 - 64,326\% = 35,674\%$$

The same procedure is applied to the three days rate and factors:

$$\text{Upward risk-neutral semiannual probability} = (1 + 0,049\% - 0,98668587) / (1,01349379 - 0,98668587) = 51,489\%$$

$$\text{Downward risk-neutral semiannual probability} = 1 - 51,489\% = 48,511\%$$

The theoretical price of the bonds is found discounting the principal and the coupons independently in a backward manner. As we can observe from the yield lattice on April 3<sup>rd</sup>, 2017 we have a total of 20 possible branches (or expected yields). For the date of October 3<sup>rd</sup>, 2008 or the date of expiration of the bond we can expect to receive a notional principal of 100 for the 21 possible branches on that date, in the same way as the principal, we can expect to receive a coupon of 4,75. As observed from the yield lattice in April 3<sup>rd</sup>, 2017 the highest yield expected in the upward branches are 14,6426% and 12,2655% respectively. Therefore, the expected principal price for those yields in April 3<sup>rd</sup>, 2017 are  $100 / (1 + 14,6426\%) = 87,2275843$  and  $100 / (1 + 12,2655\%) = 89,0745259$ . In this way we can find the expected price for the upward branch on October 3<sup>rd</sup>, 2016 by discounting the expected prices for April 3<sup>rd</sup>, 2017 and applying the risk-neutral semiannual probability for each price in the following way:

$$\text{Expected price on October 3}^{\text{rd}}, 2016 = (64,326\% \times (87,2275843 / (1 + 13,4015\%<sup>9</sup>))) + (35,674\% \times (89,0745259 / (1 + 11,2259\%))) = 78,048364$$

For the coupons the procedure is the same one used in the principal with the difference that we accrue the coupons of each period. From the yield lattice, we can observe that in April 3<sup>rd</sup>, 2017 the highest yield expected in the upward branches are 14,6426% and 12,2655% respectively. Therefore, the expected accrued coupon prices for those yields in April 3<sup>rd</sup>, 2017 are  $(4,75 / (1 + 14,6426\%)) + 4,75 = 8,89331025$  and  $(4,75 / (1 + 12,2655\%)) + 4,75 = 8,98103998$ . In this way we can find the expected accrued coupon prices for the upward branch on October 3<sup>rd</sup>, 2016 by discounting the expected

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<sup>9</sup> The yields used to discount this node are the ones in the upward branches of the yield lattice for October 3<sup>rd</sup>, 2016.

accrued coupon prices for April 3<sup>rd</sup>, 2017 and applying the risk-neutral semiannual probability for each price in the following way:

$$\text{Expected accrued coupon price on October 3}^{\text{rd}}, 2016 = (64,326\% \times (8,89331025 / (1+13,4015\%) + 4,75)) + (35,674\% \times (8,98103998 / (1+11,2259\% + 4,75))) = 12,6751827$$

In this way, we continue to value the principal backwards until April 3<sup>rd</sup>, 2008 for valuing the principal and the coupons on the date of March 31<sup>st</sup>, 2008, we use the risk three day neutral probability and the fractionate discount factor for the period (3/180=0,01666667) as follows:

$$\text{Expected price on March 31}^{\text{st}}, 2008 = (51,489\% \times (50,7393587 / (1+2,934\%)^{0,01666667})) + (48,511\% \times (56,9083069 / (1+2,934\%)^{0,01666667})) = 53,7194041$$

$$\text{Expected accrued coupon price on March 31}^{\text{st}}, 2008 = ((51,489\% \times ((70,9042734 / (1+2,934\%)^{0,01666667}) + 4,75)) + (48,511\% \times ((74,5855343 / (1+2,934\%)^{0,01666667}) + 4,75))) = 77,422509$$

The expected not callable price for the theoretical bond would be the sum of the expected price for the principal and coupons in March 31<sup>st</sup>, 2008 that means that the expected not callable price would be 53,7194041 + 77,422509 = **131,141913**. In the same way, a theoretical price can be found for each node of the not callable bond price lattice. In tables 2, 3 and 4 we can observe a summary of the results for the principal, coupons and expected bond prices:

**Table 2-Discounted Expected Principal Price Lattice**

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
53,7194041	50,7393587	50,2358917	49,706156	49,2930213	49,013601	48,8879391	48,9398023	49,1977287	49,6964246	50,47864062	51,5977141	53,12105003	55,1349379	57,75129834	61,1172515	65,42887629	70,9512843	78,048364	87,2275843	100
	56,9083069	55,8545931	55,4809313	55,0884847	54,8213314	54,6986714	54,7430495	54,9812293	55,445345	56,17442734	57,2164432	58,63104668	60,4933241	62,89894889	65,9713555	69,87184852	74,8140326	81,0847015	89,0745259	100
		61,6787896	60,7675469	60,5275903	60,2773098	60,160445	60,1979872	60,4146189	60,8396521	61,50825544	62,4630737	63,75638115	65,4529696	67,63405903	70,402648	73,89091239	78,2705599	83,7675015	90,6829197	100
			66,1870679	65,4215289	65,3138624	65,2047703	65,2362595	65,4307662	65,8146861	66,41935367	67,2823177	68,44901501	69,9749843	71,92881901	74,3961431	77,48501661	81,3333625	86,119288	92,0755986	100
				70,3968616	69,7752035	69,7940221	69,8202725	69,9930126	70,3357512	70,87613994	71,6469736	72,68748428	74,0450319	75,77732758	77,9553826	80,66745626	84,0243888	88,1668816	93,2755422	100
					74,2853417	73,801705	73,9378414	74,089836	74,392774	74,87085936	75,5525402	76,47150796	77,6679849	79,19039656	81,0975588	83,46156321	86,3716161	89,9391924	94,3050241	100
						77,8415337	77,4870513	77,7289918	77,9944956	78,41384562	79,0115793	79,8165018	80,8626667	82,19063397	83,8490943	85,89698282	88,4062508	91,4655318	95,1850332	100
							81,0644138	80,8281687	81,1629805	81,52814557	82,0485043	82,74859519	83,6571797	84,80818998	86,2419381	88,00666956	90,1605731	92,7744023	95,9349215	100
								83,9609242	83,8307862	84,24490873	84,6951528	85,30045178	86,0850523	87,07732184	88,310652	89,8246048	91,666378	93,8926904	96,5722272	100
									86,5440665	86,50738428	86,9872662	87,5079821	88,1822516	89,03378152	90,0902514	91,38416439	92,9539208	94,8451835	97,1126259	100
										88,83118026	88,875323	89,40790395	89,9849986	90,71294242	91,6147006	92,71702901	94,0512662	95,6543293	97,5699729	100
											90,8424608	90,95521822	91,5282655	92,14854223	92,9159387	93,85252237	94,9839446	96,3401723	97,9564027	100
												92,59974039	92,7695974	93,37191065	94,0233159	94,81727049	95,748293	96,9204101	98,282463	100
													94,1255245	94,34184391	94,9633307	95,63509049	96,4441669	97,4105255	98,5572659	100
														95,44223968	95,6954446	96,32703556	97,009705	97,8239636	98,7886428	100
															96,5715341	96,85360811	97,4868787	98,1723281	98,9832956	100
																97,53306129	97,8394959	98,4655838	99,1469398	100
																	98,3407144	98,6778103	99,284435	100
																		98,9892723	99,399903	100
																			99,4078454	100
																				100

**Table 3-Discounted Coupon Expected Price Lattice**

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
77,422509	70,9042734	67,0769206	63,1922705	59,4030284	55,7084134	52,1066223	48,5946552	45,1681019	41,8208699	38,54483448	35,3293782	32,16077589	29,0213589	25,88836391	22,7323229	19,51477933	16,1850019	12,6751827	8,89331025	4,75
	74,5855343	69,9772821	66,1028815	62,1767708	58,3357035	54,5782624	50,9019038	47,3027505	43,7753338	40,31226825	36,9038306	33,53741008	30,1967767	26,86109345	23,5035653	20,08956327	16,5739853	12,8974897	8,98103998	4,75
		73,17616	68,5997742	64,67612	60,7056275	56,8096779	52,9861277	49,2315964	45,5412266	41,90838513	38,3242843	34,77749502	31,2533116	27,73291121	24,1922261	20,60041035	16,9177276	13,0926011	9,05743869	4,75
			71,3475219	66,795754	62,8203815	58,8023084	54,8482723	50,955346	47,1192538	43,33410019	39,5920331	35,88281624	32,1932821	28,50661993	24,801439	21,05051859	17,2191197	13,2626757	9,12359093	4,75
				69,1288899	64,5943234	60,5648777	56,4961545	52,48109	48,515949	44,59553905	40,7129123	36,85899388	33,0221128	29,18740191	25,336021	21,44414002	17,4815892	13,4100488	9,18058826	4,75
					66,5524467	62,0279016	57,9424176	53,8204135	49,7419475	45,70248133	41,6959189	37,71428507	33,7473248	29,78199943	25,8018435	21,786137	17,7088259	13,5370988	9,22948864	4,75
						63,6518112	59,1306416	54,9877339	50,8104674	46,66699571	42,5520086	38,45854907	34,376733	30,29802729	26,2053159	22,08162779	17,9045705	13,646148	9,27128908	4,75
							60,4608043	55,9370985	51,7360685	47,50232645	43,2931181	39,10241317	34,9224603	30,74343258	26,5529884	22,33572079	18,0724622	13,7393956	9,30690877	4,75
								57,0124734	52,4811677	48,22204071	43,9314208	39,65664119	35,3910201	31,12609603	26,8512644	22,55332793	18,2159356	13,8188763	9,33718079	4,75
									53,3383617	48,79528406	44,4787929	40,13168504	35,7923577	31,45355684	27,1062072	22,73904398	18,3381594	13,8864381	9,36284973	4,75
										49,46799986	44,9098605	40,53738914	36,1349143	31,73283856	27,3234217	22,89707757	18,4420051	13,9437359	9,38457371	4,75
											45,4285873	40,85293034	36,4264323	31,97035287	27,5079948	23,03122075	18,530038	13,9922341	9,40292913	4,75
												41,2448271	36,6499576	32,17186084	27,6644757	23,14484582	18,6045246	14,0332161	9,41841699	4,75
													36,9388825	32,32378028	27,7968858	23,24092039	18,6674483	14,0677985	9,43147013	4,75
														32,53042029	27,8946527	23,32203379	18,7205323	14,0969464	9,44246053	4,75
															28,0367096	23,3803627	18,7652645	14,1214894	9,45170654	4,75
																23,47273059	18,7964055	14,1421379	9,45947964	4,75
																	18,8511976	14,1562157	9,46601066	4,75
																		14,1822323	9,47149539	4,75
																			9,47187266	4,75
																				4,75



**Table 4-Theoretical Expected Not Callable Bond Prices-(The Sum of Table 2 and 3 for each node)**

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
131,141913	121,643632	117,312812	112,898426	108,69605	104,722014	100,994561	97,5344574	94,3658305	91,5172945	89,0234751	86,9270924	85,28182591	84,1562968	83,63966225	83,8495744	84,94365562	87,1362862	90,7235467	96,1208946	104,75
	131,493841	125,831875	121,583813	117,265256	113,157035	109,276934	105,644953	102,28398	99,2206788	96,4866956	94,1202738	92,16845676	90,6901008	89,76004234	89,4749209	89,96141179	91,3880178	93,9821911	98,0555659	104,75
		134,85495	129,367321	125,20371	120,982937	116,970123	113,184115	109,646215	106,380879	103,4166406	100,787358	98,53387617	96,7062812	95,36697025	94,5948741	94,49132274	95,1882875	96,8601026	99,7403584	104,75
			137,53459	132,217283	128,134244	124,007079	120,084532	116,386112	112,93394	109,7534539	106,874351	104,3318313	102,168266	100,4354389	99,1975822	98,5355352	98,5524823	99,3819636	101,19919	104,75
				139,525751	134,369527	130,3589	126,316427	122,474103	118,8517	115,471679	112,359886	109,5464782	107,067145	104,9647295	103,291404	102,1115963	101,505978	101,57693	102,45613	104,75
					140,837788	135,829607	131,880259	127,91025	124,134722	120,5733407	117,248459	114,185793	111,41531	108,972396	106,899402	105,2477002	104,080442	103,476291	103,534513	104,75
						141,493345	136,617693	132,716726	128,804963	125,0808413	121,563588	118,2750509	115,24034	112,4886613	110,05441	107,9786106	106,310821	105,11168	104,456322	104,75
							141,525218	136,765267	132,899049	129,030472	125,341622	121,8510084	118,57964	115,5516226	112,794927	110,3423904	108,233035	106,513798	105,24183	104,75
								140,973398	136,311954	132,4669494	128,626574	124,957093	121,476072	118,2034179	115,161916	112,3779327	109,882314	107,711567	105,909408	104,75
									139,882428	135,3026683	131,466059	127,6396671	123,974609	120,4873384	117,196459	114,1232084	111,29208	108,731622	106,475476	104,75
										138,2991801	133,785183	129,9452931	126,119913	122,445781	118,938122	115,6141066	112,493271	109,598065	106,954547	104,75
											136,271048	131,8081486	127,954698	124,1188951	120,423934	116,8837431	113,513983	110,332406	107,359332	104,75
												133,8445675	129,419555	125,5437715	121,687792	117,9621163	114,379354	110,953626	107,70088	104,75
													131,064407	126,6656242	122,760217	118,8760109	115,111615	111,478324	107,988736	104,75
														127,97266	123,590097	119,6490694	115,730237	111,92091	108,231103	104,75
															124,608244	120,2339708	116,252143	112,293818	108,435002	104,75
																121,0057919	116,635901	112,607722	108,606419	104,75
																	117,191912	112,834026	108,750446	104,75
																		113,171505	108,871398	104,75
																			108,879718	104,75
																				104,75

**Step 5-Finding a theoretical call price for each option embedded in the callable bond using the theoretical not callable sovereign bond price lattice.**

Once we have the expected not callable price for each node until maturity we can proceed to calculate the theoretical value for each option embedded on the bond according to the following call schedule:

Date (mm/dd/yyyy)	Exercise Price
10/03/2012	104,75
10/03/2013	103,167
10/03/2014	101,583
10/03/2015	100

Since the call is priced backwards we begin with the first option that has an exercise price of 104,75 on October 3<sup>rd</sup>, 2012. As we can appreciate from the not callable bond price lattice from the possible 11 expected prices on October 3<sup>rd</sup>, 2012, just 8 of them will be in the money or have an exercise price that is greater than the expected price. Therefore, the possible notional call prices on that date would be as follow:

<b>03/10/2012</b>
9
0
0
0
5,00345386
10,72167899
15,82334068
20,33084133
24,28047202
27,71694944
30,55266834
33,54918012

If the exercise price is 104,75 and the expected price on the upward node is 89,0234751, then the call price would be zero because  $C = \text{MAX}(0, 89,0234751 - 104,75)$ . In the case of the fourth node, the call price would be 5,00345386 because  $C = \text{MAX}(0, 109,7534539 - 104,75)$  and so forth until the call price for each node for an expected not callable price is found. Then the call option is priced backwards using the semiannual risk-neutral probability in the following manner:

$$\text{Expected Call Price Fourth Node on April 3}^{rd}, 2012 = ((51,489\% \times 5,00345386) + (48,511\% \times 10,72167899)) / (1 + 3,5494\%^{10}) = 6,80192536$$

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<sup>10</sup> This is the yield found in the fourth node on April 3<sup>rd</sup>, 2012

Then we continue to price the call backwards until April 3<sup>rd</sup>, 2008. For valuing the call option on March 31<sup>st</sup>, 2008, we use the risk three day neutral probability and the fractionate discount factor for the period (3/180=0,01666667) as follows:

$$\text{Expected Call Price on March 31}^{\text{st}}, 2008 = ((51,489\% \times 5,35010251) + (48,511\% \times 9,06234989)) / (1 + 2,934\%^{11})^{0,01666667} = 7,14751384$$

It is important to notice that in the nodes that the option is exercised, for the next option just the nodes that were not exercised in the first option will be taken into account when valuing the second option scheduled on October 3<sup>rd</sup>, 2013. Therefore, the expected prices used to price the second option would be (notice that the following paths after the exercise of the first option cease to exist because the bond has been recalled by the issuer through the exercise of the first call option):

03/10/2012	03/04/2013	03/10/2013
9	10	11

89,0234751	86,9270924	85,2818259
96,4866956	94,1202738	92,1684568
103,416641	100,787358	98,5338762
Exercise	106,874351	104,331831
Exercise	0	109,546478
Exercise	0	0
Exercise	0	0
Exercise	0	0
Exercise	0	0
Exercise	0	0
Exercise	0	0
Exercise	0	0
	0	0
		0

If the second option exercise price on October 3<sup>rd</sup>, 2013 is 103,167 and we just have 5 expected prices for that date the notional call prices for the second option would be:

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<sup>11</sup> This is the yield found in the first node on March 31<sup>st</sup>, 2008 in the yield lattice.

03/10/2013
11

0
0
0
1,164831251
6,379478166
0
0
0
0
0
0
0
0
0

If the stated price for that date is greater than the exercise price of the option of 103,167, the option will be exercised otherwise the option will expire out of the money and its value would be zero. With these notional call values, we use the same procedure of the first option to find the value of the second option on March 31<sup>st</sup>, 2008. The third and fourth option call values are found in the same way as the second option (just taking into account the stated prices that have not been exercised in the previous option until the last option expires). The results for the four options are shown in tables 5 to 8:

**Table 5-First call lattice 10/03/2012 Strike 104,75**

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017	
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
7,14751384	5,35010251	4,21442342	3,13878823	2,1563276	1,30821157	0,64102035	0,19824583	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	9,06234989	7,84384789	6,53777557	5,22264937	3,92005799	2,66667152	1,52261755	0,58383132	0	0	0	0	0	0	0	0	0	0	0	0	0
		12,0047814	10,7972721	9,45408413	8,04705873	6,57018326	5,0195138	3,39628116	1,71235821	0	0	0	0	0	0	0	0	0	0	0	0
			15,0856837	13,9730989	12,7123751	11,3808225	9,96463518	8,44593374	6,80192536	5,00345386	0	0	0	0	0	0	0	0	0	0	0
				18,1310618	17,1392588	16,0009235	14,8026066	13,5336394	12,179506	10,72167899	0	0	0	0	0	0	0	0	0	0	0
					21,062564	20,1942103	19,1841239	18,124231	17,0077463	15,82334068	0	0	0	0	0	0	0	0	0	0	0
						23,8437271	23,0964587	22,2172145	21,2955458	20,33084133	0	0	0	0	0	0	0	0	0	0	0
							26,4504107	25,8133294	25,0683085	24,28047202	0	0	0	0	0	0	0	0	0	0	0
								28,8776654	28,3140783	27,71694944	0	0	0	0	0	0	0	0	0	0	0
									31,1714061	30,55266834	0	0	0	0	0	0	0	0	0	0	0
										33,54918012	0	0	0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0	0	0
													0	0	0	0	0	0	0	0	0
														0	0	0	0	0	0	0	0
															0	0	0	0	0	0	0
																0	0	0	0	0	0
																	0	0	0	0	0
																		0	0	0	0
																			0	0	0
																				0	0
																					0

**Table 6-Second call lattice 10/03/2013 Strike 103,167**

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017	
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
0,46622344	0,55532809	0,62424349	0,67228019	0,68439849	0,64507164	0,54372273	0,38411435	0,19659165	0,04498284	0	0	0	0	0	0	0	0	0	0	0	0
	0,37211292	0,47734457	0,59447031	0,71731877	0,82971454	0,9044444	0,9020912	0,77672022	0,50042909	0,133710036	0	0	0	0	0	0	0	0	0	0	0
		0,21296143	0,30255705	0,42249773	0,57996995	0,77751713	1,00701172	1,23530908	1,37572804	1,232652873	0,39553022	0	0	0	0	0	0	0	0	0	0
			0,06743232	0,10740455	0,17077425	0,27209311	0,43449921	0,69554403	1,11640059	1,797128571	2,90211586	1,164831251	0	0	0	0	0	0	0	0	0
				0	0	0	0	0	0	0	0	6,379478166	0	0	0	0	0	0	0	0	0
					0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
								0	0	0	0	0	0	0	0	0	0	0	0	0	0
									0	0	0	0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0	0	0
													0	0	0	0	0	0	0	0	0
														0	0	0	0	0	0	0	0
															0	0	0	0	0	0	0
																0	0	0	0	0	0
																	0	0	0	0	0
																		0	0	0	0
																			0	0	0
																				0	0
																					0

**Table 7-Third call lattice 10/03/2014 Strike 101,583**

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0,41275689	0,48959321	0,53673782	0,56751723	0,57375719	0,54792176	0,48545111	0,38792313	0,26632157	0,14256505	0,045914756	0	0	0	0	0	0	0	0	0	0
	0,33120445	0,40458269	0,48123681	0,55626541	0,62034328	0,660568	0,66131224	0,60719344	0,48947766	0,316843558	0,12870763	0	0	0	0	0	0	0	0	0
		0,22612651	0,30008093	0,38976947	0,49606804	0,61524944	0,73768842	0,84472416	0,90555708	0,87660521	0,70972344	0,384597115	0	0	0	0	0	0	0	0
			0,11136847	0,16124606	0,23057864	0,32633554	0,45568485	0,62487869	0,8353676	1,075286605	1,30182691	1,405946176	1,14320258	0	0	0	0	0	0	0
				0,02981199	0,04748382	0,07549972	0,12029304	0,19209317	0,30750173	0,493563453	0,79451497	1,283032577	2,07908525	3,381729488	0	0	0	0	0	0
					0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0	0	0	0	0	0	0
								0	0	0	0	0	0	0	0	0	0	0	0	0
									0	0	0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0	0
													0	0	0	0	0	0	0	0
														0	0	0	0	0	0	0
															0	0	0	0	0	0
																0	0	0	0	0
																	0	0	0	0
																		0	0	0
																			0	0
																				0

**Table 8-Fourth call lattice 10/03/2015 Strike 100**

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0,34654265	0,43377787	0,49720683	0,55474064	0,59919007	0,62203725	0,614513	0,5695926	0,48499106	0,36677309	0,232242182	0,10926999	0,027739104	0	0	0	0	0	0	0	0
	0,25395304	0,31940361	0,39346265	0,47459003	0,55799233	0,63560488	0,69551287	0,72214496	0,69816011	0,60935741	0,45398399	0,256285388	0,07775788	0	0	0	0	0	0	0
		0,15681735	0,21248203	0,28300503	0,37142061	0,47870294	0,60307691	0,73775445	0,86779438	0,96645571	0,99267751	0,894439831	0,6299961	0,235139142	0	0	0	0	0	0
			0,06933946	0,10152413	0,14716228	0,21179351	0,30206316	0,42578997	0,59089885	0,802512637	1,05621645	1,323616507	1,52269318	1,458520265	0,70665779	0	0	0	0	0
				0,0165233	0,02631791	0,04184572	0,06667242	0,10646765	0,17043285	0,273557571	0,44035997	0,711120877	1,15233312	1,874323757	3,06116884	2,111596285	0	0	0	0
					0	0	0	0	0	0	0	0	0	0	0	5,247700211	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0	0	0	0	0	0	0
								0	0	0	0	0	0	0	0	0	0	0	0	0
									0	0	0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0	0
													0	0	0	0	0	0	0	0
														0	0	0	0	0	0	0
															0	0	0	0	0	0
																0	0	0	0	0
																	0	0	0	0
																		0	0	0
																			0	0
																				0



Therefore, by adding the four option call prices we found that the embedded options of the bond have a total value of  $7,14751384 + 0,46622344 + 0,41275689 + 0,34654265 = 8,37303681$ .

**Step 6-Finding the theoretical option adjusted spread for a theoretical Colombian sovereign not callable bond.**

Since we know that the theoretical dirty price of a Colombian Sovereign Bond with a coupon of 9,5% in March 31<sup>st</sup>, 2008 is 131,141913. Also, we know that the value of the call option in the hands of the issuer is 8,37303681. Therefore, the expected dirty price in March 31<sup>st</sup>, 2008 of a theoretical callable Colombian sovereign bond with the same maturity, coupon and call schedule as TGI would be  $131,141913 - 8,37303681 = 122,768876$ . If the bond pays a 4,75% semiannual coupon on April 3<sup>rd</sup>, 2008 on a 30/360 basis then the accrued interest until that date would be  $((4,5\%/180) \times 177) = 4,67083333$ . Therefore, the clean price of our theoretical callable bond would be  $122,768876 - 4,67083333 = 118,098043$  and the expected yield of a theoretical sovereign Colombian callable bond would be:

Liquidation date:	31/03/2008
Settlement:	03/10/2017
Coupon:	9,50%
Principal:	100
Clean price:	118,098043
Yield:	<b>6,875%</b>

If we know that the spread of a Theoretical not callable Colombian Sovereign Bond on March 31<sup>st</sup>, 2008 is 5,867% then  $6,875\% - 5,867\% = 1,008\%$  or approximately 100,8 basis points are attributable to the value of the call options that the investor in theory “sells” to the issuer which is the value of the OAS in this specific example. Similarly, if we know that in March 31<sup>st</sup>, 2008 the market yield of TGI is 8,872%, and we already know the theoretical OAS for a Theoretical Colombian Sovereign Bond, then we can assume that the difference in spread can be attributable to the company specific risk of a natural gas company operating in Colombia. In this case this risk can be valued as an additional spread of  $8,872\% - 6,875\% = 1,997\%$  or approximately 199,7 basis points. For investment strategy purposes, if we can assume that the company specific risk is constant and that changes in yield are attributable to the country risk and the OAS of the bond on a following date, then we can verify if the callable bond is overpriced or underpriced on that date depending on the expected theoretical OAS or country risk variation.

## **CONCLUSIONS**

This paper presents a complete detailed methodological approach for valuing callable bonds in Emerging Markets. Through the development of a practical example using the binomial pricing model, it was possible to determine what would be the theoretical value of the Option Adjusted Spread of TGI. Moreover, by using meaningful proxy variables taken from real life data, it is possible to find better estimates of the spread attributable to specific risk of companies operating in emerging markets. Also, of special importance is the determination of a theoretical sovereign price for a bond that has the same country of origin as the company whose callable bond issue we wish to value. Finally, by applying a commonly used methodology such as the binomial pricing formula, we expect to set the grounds for further research on how to develop methodological approaches on how find meaningful proxy variables for complex valuation models using real market data.

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## **DATA SOURCE:**

BLOOMBERG